Parameter estimation in layered transversely isotropic media using reflection moveout of converted waves

Ilya Tsvankin and Vladimir Grechka
Center for Wave Phenomena

ABSTRACT

P-wave reflection moveout in stratified transversely isotropic media with a vertical symmetry axis (VTI) can be used to determine velocity models in the time domain and carry out time imaging. Additional data (e.g., borehole information about vertical velocity), however, are needed to constrain reflector depth and Thomsen's anisotropy parameters ϵ and δ required for depth migration. Here, extending our previous results for a single VTI layer, we show that the interval values of ϵ, δ and the vertical velocities of the P- and S-waves can be found from surface data alone by combining P-wave moveout with the traveltimes of the mode-converted PSV-wave reflected from horizontal and dipping interfaces.

At the first stage of the velocity-analysis procedure, we use common-midpoint (CMP) moveout of P- and PSV-waves to build an initial anisotropic velocity model. As in the single-layer problem, stable parameter estimates cannot be obtained without including the moveout of converted waves from a *dipping* interface. The CMP traveltime and offset of the PSV-wave in layered VTI media above a dipping reflector can be conveniently expressed in terms of the interval slowness components of P- and S-waves. This analytic representation provides an efficient way of incorporating the slope of the PSV moveout curve at zero offset and short-offset traveltimes as a whole into the inversion algorithm.

The main shortcoming of operating with converted-wave moveout in CMP geometry is the distorting influence of reflection-point dispersal. To overcome this problem, at the second stage of the parameter-estimation procedure we resort the PSV data into common-reflection-point (CRP) gathers using the obtained interval parameters of the VTI model. Then, without changing the P-wave normal-moveout velocities, we update the medium parameters to match the short-offset PSV traveltimes on CRP gathers from dipping reflectors. The inversion yields the depth-dependent VTI model suitable for both depth migration of P-wave data and processing (e.g., transformation to zero offset) of converted modes.

The analytic formalism and approach to depth-domain velocity analysis described here can be used not just for vertical transverse isotropy, but also for vertical symmetry planes in lower-symmetry vertically inhomogeneous media, such as orthorhombic and monoclinic. Furthermore, the results are applicable to a wide class of models with weak azimuthal anisotropy where out-of-plane phenomena in the dip plane of the reflector can be neglected. A complete parameter-estimation procedure in azimuthally anisotropic media, however, should be performed in 3-D geometry using a wide range of source-receiver azimuths.

Introduction

With recent advances in the acquisition of multicomponent data, such as the new technology of ocean bottom surveys, converted waves find an increasing number of applications in seismic exploration. For example, PS-waves help in imaging hydrocarbon reservoirs beneath gas clouds, where conventional P-wave methods fail due to the high attenuation of compressional energy (Thomsen, 1999). Also, converted waves provide information about shear-wave velocities and other medium parameters which cannot be constrained using P-wave data alone. This advantage of mode conversions becomes especially important in anisotropic media due to the multi-parameter nature of the inverse problem and the ambiguity in estimating reflector depth from surface P-wave data.

For the transversely isotropic model with a vertical symmetry axis (VTI medium), P-wave reflection traveltimes alone are generally insufficient to determine reflector depth (or vertical velocity) and establish the depth scale of the model. If the medium consists of horizontal VTI layers above a dipping reflector, P-wave moveout is fully controlled by the interval values of the normal-moveout (NMO) velocity from a horizontal reflector $[V_{\text{nmo},P}(0)]$ and the "anellipticity" parameter η (Alkhalifah and Tsvankin, 1995; Grechka and Tsvankin, 1998b). In terms of the P-wave vertical velocity V_{P0} and Thomsen's (1986) anisotropic coefficients ϵ and δ (the three parameters that control P-wave kinematics; see Tsvankin, 1996), the two Alkhalifah-Tsvankin parameters can be expressed as

$$V_{\text{nmo},P}(0) = V_{P0}\sqrt{1+2\delta}\,, \qquad (1)$$

$$\eta = \frac{\epsilon-\delta}{1+2\delta}\,. \qquad (2)$$

The parameters $V_{\text{nmo},P}(0)$ and η can be found from the dip dependence of the P-wave NMO velocity or non-hyperbolic (long-spread) moveout of horizontal events and used for all time-domain P-wave processing steps including NMO, dip-moveout (DMO) correction and time migration (Alkhalifah and Tsvankin 1995; Grechka and Tsvankin 1998a,b). While this time-processing methodology has proved to be quite effective on field data (Alkhalifah et al., 1996; Anderson et al., 1996), the inherent trade-offs between V_{P0}, ϵ and δ in equations (1) and (2) precludes anisotropic parameter estimation in *depth*.

To construct anisotropic models for depth imaging, P-wave reflection traveltimes have to be combined with borehole information (e.g., check shots or well logs), shear or converted waves. In the exploration context, the most practical option is the joint inversion of P- and PS-reflections, particularly for offshore surveys with data collection on the ocean bottom (OBS). Tsvankin and Grechka (1999; hereafter referred to as Paper I) examined this inverse problem for the model of a single VTI layer. Their analysis shows that it is not sufficient to supplement P-wave NMO velocities from horizontal and dipping reflectors (yielding the parameters $V_{\text{nmo},P}(0)$ and η) with the traveltimes of horizontal PSV events (the PSV-wave will be denoted here simply by PS). To achieve sufficient stability in estimating the vertical velocities, the inversion procedure has to include *dip moveout* (i.e., reflection traveltimes from a dipping interface) of PS-waves.

If the reflector is dipping, the common-midpoint moveout curve of the PS-wave is asymmetric with respect to zero offset (i.e., the traveltime does not stay the same if the source and receiver are interchanged) and may not have a minimum for relatively steep dips exceeding 40-50°. Paper I introduces an exact parametric representation of PS-wave moveout on CMP gathers in VTI media and gives concise expressions for such attributes of the traveltime curve as the slope at zero offset, NMO velocity at the traveltime minimum etc. These attributes of dipping PS events have proved to be sufficiently sensitive to the VTI parameters for the joint inversion of P- and PS traveltimes to give accurate estimates of the P- and S-wave vertical velocities V_{P0} and V_{S0} and the anisotropic coefficients ϵ and δ. The main limitations of the algorithm developed in Paper I are the simplicity of the model (single layer) and reliance on CMP geometry in which converted modes suffer from reflection-point dispersal.

Here, we extend the methodology of Paper I to the more realistic vertically inhomogeneous VTI media above a dipping reflector. Assuming that the acquisition line is parallel to the dip plane of the reflector, we derive parametric 2-D expressions for the moveout of the PS-wave valid for both common-midpoint and common-reflection-point (CRP) gathers. These equations, which can be used in any anisotropic medium with a vertical symmetry plane, provide a basis for the VTI inversion algorithm. First, we use dip moveout of PS-waves on CMP gathers (in combination with the NMO velocity of horizontal PS events and P-wave data) to obtain an approximate anisotropic model that may be distorted by reflection-point dispersal. Then the model is refined by resorting the PS reflections from both horizontal and dipping interfaces into CRP gathers and repeating the parameter-estimation procedure. Numerical examples confirm the accuracy and efficiency of our algorithm for typical multilayered VTI models.

Moveout of PS-waves in symmetry planes of layered anisotropic media

Conventional-spread reflection moveout of pure modes on CMP gathers in both isotropic and anisotropic media is usually close to a hyperbolic curve parameterized by normal-moveout velocity (e.g., Tsvankin and Thomsen, 1994; Grechka and Tsvankin, 1998b). For mode-converted waves, however, CMP moveout is generally asymmetric with respect to zero offset and cannot be fitted to a hyperbola centered at the CMP location. Only in the special case of horizontally layered media with a horizontal symmetry plane the traveltime of converted

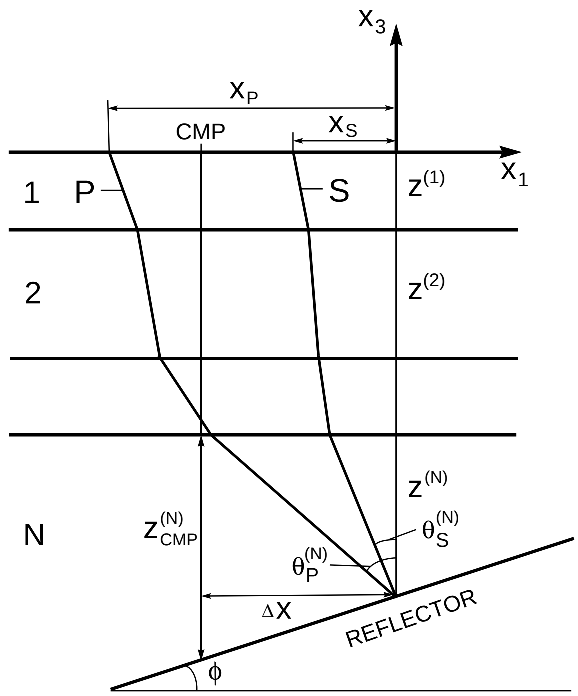

Figure 1. Geometry of the PS-reflection from a dipping interface in a vertical symmetry plane of layered anisotropic media. x_P and x_S are the horizontal displacements of the P- and S-rays with respect to the reflection point; $z^{(\mathrm{N})}$ and $z^{(\mathrm{N})}_{\mathrm{CMP}}$ are the thicknesses of layer N measured above the reflection point and below the common midpoint (CMP), respectively.

waves becomes an even function of the source-receiver offset (Grechka, Theophanis, and Tsvankin, 1999).

Here, we extend the methodology of Paper I to give a parametric representation of reflection moveout of converted waves in vertical symmetry planes of layered anisotropic media. The acquisition line is confined to the dip plane of a reflector overlaid by an arbitrary number of horizontal layers (Figure 1). The anisotropic symmetry does not need to be specified at this stage, but the vertical incidence plane is assumed to be a plane of mirror symmetry in all layers. Therefore, both rays and phase-velocity vectors of reflected waves cannot deviate from the dip plane of the reflector, and the kinematics (but not necessarily dynamics) of wave propagation can be treated in two dimensions. First, we present general expressions for the traveltime and offset on both common-reflection-point (CRP) and common-midpoint (CMP) gathers and then show how to obtain the moveout attributes needed in the inversion procedure.

Traveltime and offset on CRP and CMP gathers

Figure 2 illustrates the behavior of converted-wave moveout on both CMP and CRP gathers above a layered VTI medium. Similar to the single-layer case discussed in Paper I, the offset of the traveltime minimum on CMP gathers increases with dip, which makes the moveout curve increasingly asymmetric. If the dip does not exceed 30-40°, the traveltime minimum can often be recorded on a sufficiently long CMP gather (Figure 2a). For steeper dips the traveltime monotonically decreases with offset (Figure 2c,e), and the short-offset moveout is largely controlled by the slope of the moveout curve at zero offset. This shape of the traveltime function suggests that the moveout attributes of the PS-wave suitable for the parameter-estimation procedure can include the zero-offset moveout slope $dt/dx|_{x=0}$ and, for mild dips, the normalized offset of the traveltime minimum $x_{\min}/t_{\min}$. Another attribute that proved useful in the single-layer model is the NMO velocity of the PS-wave at the traveltime minimum (Paper I), but for multilayered VTI media is difficult to derive it in a closed analytic form.

The dependence of converted-wave CRP moveout on dip is more complicated. For the model in Figure 2, the traveltime minimum first moves towards negative offsets with increasing dip (Figure 2b,d) and then returns back almost to the zero-offset location (Figure 2f).

For CRP gathers, the traveltime and offset of the PS-wave can be found by summing up the single-layer solutions of Paper I. As shown in Appendix A, the traveltime of the PS arrival with a reflection (conversion) point an the N-th interface (Figure 1) is given by

$$t = \sum_{\ell=1}^{N} z^{(\ell)} \left(q_P^{(\ell)} - p_P q_{,P}^{(\ell)} + q_S^{(\ell)} - p_S q_{,S}^{(\ell)} \right), \qquad (3)$$

where $z^{(\ell)}$ ($\ell = 1, 2, \dots N-1$) are the thicknesses of the horizontal layers in the overburden, $z^{(\mathrm{N})}$ is the thickness of layer N above the reflection point, $q_P^{(\ell)}$ and $q_S^{(\ell)}$ are the interval vertical slownesses of the P- and S-waves, p_P and p_S are the horizontal slownesses (ray parameters), $q_{,P}^{(\ell)} \equiv dq_P^{(\ell)}/dp_P$ and $q_{,S}^{(\ell)} \equiv dq_S^{(\ell)}/dp_S$. Since the medium above the reflector is laterally homogeneous, the ray parameters p_P and p_S remain constant between the reflector and the surface; they are related to each other through Snell's law at the reflector.

The corresponding source-receiver offset is obtained in Appendix A as

$$x = x_S - x_P = \sum_{\ell=1}^{N} z^{(\ell)} \left(q_{,P}^{(\ell)} - q_{,S}^{(\ell)} \right). \qquad (4)$$

Since the x_1-axis points updip (Figure 1), the offset x in equation (4) is positive if the P-leg is located *downdip* with respect to the S-leg; the same sign convention was adopted in Paper I.

To compute the traveltime and offset from equations (3) and (4), we need to specify one of the ray parameters (p_P or p_S) and find the other from Snell's law at the reflector. The vertical slownesses $q_P^{(\ell)}$ and $q_S^{(\ell)}$ in

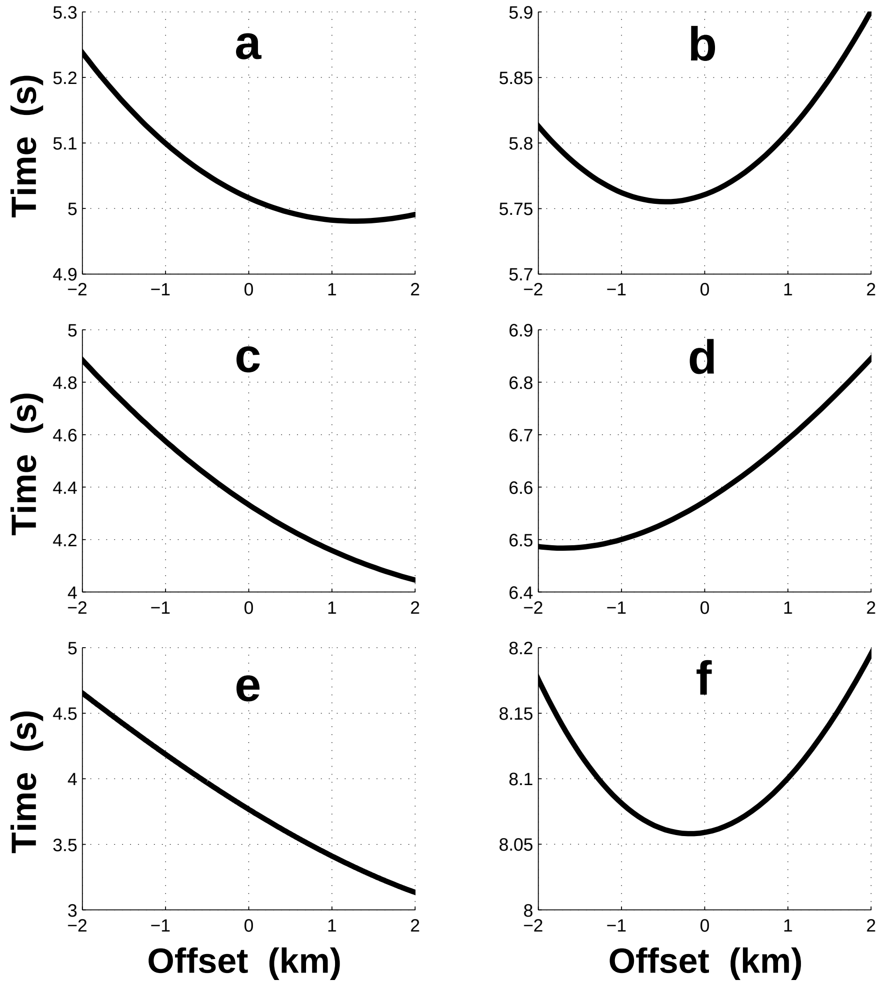

Figure 2. Dip-line moveout of the PSV-wave reflected from (i.e., converted at) a dipping interface beneath three VTI layers (the top two layers are horizontal). The left column (a,c,e) are the common-midpoint (CMP) gathers, the right column (b,d,f) are the common-reflection-point (CRP) gathers. Each row corresponds to a different reflector dip ϕ: $\phi = 20°$ (a,b), $\phi = 40°$ (c,d) and $\phi = 60°$ (e,f). Positive offsets correspond to the P-wave leg located downdip from the S-wave leg.
The top layer (Layer 1) has the following parameters: $V_{P0}^{(1)} = 2.0$ km/s, $V_{S0}^{(1)} = 0.8$ km/s, $\epsilon^{(1)} = 0.1$, $\delta^{(1)} = 0.05$, the thickness $z^{(1)} = 0.5$ km; for Layer 2, $V_{P0}^{(2)} = 2.3$ km/s, $V_{S0}^{(2)} = 1.0$ km/s, $\epsilon^{(2)} = 0.2$, $\delta^{(2)} = 0.1$, $z^{(2)} = 1.5$ km; for Layer 3, $V_{P0}^{(3)} = 2.9$ km/s, $V_{S0}^{(3)} = 1.2$ km/s, $\epsilon^{(3)} = 0.15$, $\delta^{(3)} = 0.1$, $z^{(3)} = 2$ km ($z^{(3)}$ is the thickness of Layer 3 beneath the CMP or above the CRP).

each layer, along with the derivatives $q_{,P}^{(\ell)}$ and $q_{,S}^{(\ell)}$, can be determined from the Christoffel equation (Grechka, Tsvankin, and Cohen, 1999). Therefore, scanning over one of the ray parameters produces the CRP gather for a reflection point located at the depth $\sum_{\ell=1}^{N} z^{(\ell)}$.

To generate a CMP gather, it is necessary to relate the thickness $z^{(\mathrm{N})}$ of the N-th layer above the CRP in equations (3) and (4) to the corresponding thickness $z_{\mathrm{CMP}}^{(\mathrm{N})}$ beneath the common midpoint (Figure 1). The derivation in Appendix A leads to

$$z^{(\mathrm{N})} = \frac{z_{\mathrm{CMP}}^{(\mathrm{N})} - \frac{\tan\phi}{2} \sum_{\ell=1}^{N-1} \left[z^{(\ell)} \left(q_{,P}^{(\ell)} + q_{,S}^{(\ell)}\right)\right]}{1 + \frac{\tan\phi}{2} \left(q_{,P}^{(\mathrm{N})} + q_{,S}^{(\mathrm{N})}\right)} \,. \qquad (5)$$

Substitution of $z^{(\mathrm{N})}$ from equation (5) into equations (3) and (4) yields the traveltime and offset of the PS-wave recorded on the CMP gather specified by $z_{\mathrm{CMP}}^{(\mathrm{N})}$. In the special case of a single layer, the sum over the $N-1$ layers in the numerator of equation (5) has to be dropped, and $z^{(\mathrm{N})}$ reduces to the expression obtained in Paper I.

Equations (3) – (5) make it possible to compute a CMP gather of converted waves in symmetry planes of layered anisotropic media without time-consuming two-point ray tracing. It is still necessary to satisfy Snell's law at the reflector and solve the Christoffel equation in each layer for both P- and S-waves, but the whole computation of t and x for a given source-receiver pair has to be performed only once (i.e., just for one ray). Also, note that in VTI media the Christoffel equation $q(p) = 0$ has an analytic solution because it reduces to a quadratic polynomial in q^2.

Attributes of the moveout curve

Moveout slope at zero offset

Since the CMP traveltime of converted waves is generally asymmetric with respect to $x = 0$, short-spread moveout is largely controlled by the slope of the moveout curve at zero offset (rather than by the NMO velocity, as is the case for pure modes). The results of Paper I show that the zero-offset moveout slope of the PS-wave in a VTI layer is quite sensitive to the anisotropic parameters and represents a useful attribute for moveout inversion.

As proved in Paper I, the slope (apparent slowness) of the CMP moveout curve of any pure or converted wave recorded in a vertical symmetry plane is equal to one half of the difference between the ray parameters (horizontal slownesses) at the source and receiver locations. This result is not limited to a single layer and can be used for 2-D wave propagation in laterally and vertically inhomogeneous anisotropic media. Taking into account that the horizontal coordinate axis runs updip while the P-wave leg is located downdip from the S-leg (Figure 1), the slope of the common-midpoint PS moveout can be written as (Paper I)

$$\frac{dt}{dx} = \frac{1}{2}\left(p_S - p_P\right), \tag{6}$$

where the horizontal slownesses p_P and p_S are measured at the source and receiver locations. In our model, the medium above the reflector is horizontally homogeneous, and each ray parameter does not change between the reflector and the surface.

To obtain the moveout slope at $x = 0$, we need to compute the values of p_P and p_S for the zero-offset PS ray. The source-receiver offset in CMP geometry can be found from equation (4) with $z^{(\mathrm{N})}$ defined in equation (5). Setting the offset x in equation (4) to zero yields an equation that can be solved for one of the ray parameters (e.g., p_P); the other ray parameter is determined from Snell's law at the reflector.

Normalized offset of the traveltime minimum

For relatively mild dips up to 30-40°, the common-midpoint traveltime curve of the PS-wave usually has a minimum ($t_{\min}$) at a certain offset $x_{\min}$ (Figure 2). In the single-layer model, the ratio $x_{\min}/t_{\min}$ proved to be rather sensitive to the anisotropic parameters (Paper I). For stratified VTI media, the values of $x_{\min}$ and $t_{\min}$ can be found from equations (3), (4) and (5), where the ray parameters $p_P^{\min}$ and $p_S^{\min}$ should correspond to the traveltime minimum. Since at $x = x_{\min}$ the slope of the moveout curve goes to zero, the ray parameters should be equal to each other: $p_P^{\min} = p_S^{\min} = p^{\min}$ [see equation (6)]. Hence, the ratio $x_{\min}/t_{\min}$ is given by

$$\frac{x_{\min}}{t_{\min}} = \frac{\sum_{\ell=1}^{N} z^{(\ell)}\,(q_{,P}^{(\ell)} - q_{,S}^{(\ell)})}{\sum_{\ell=1}^{N} z^{(\ell)}\,(q_P^{(\ell)} - p^{\min} q_{,P}^{(\ell)} + q_S^{(\ell)} - p^{\min} q_{,S}^{(\ell)})}\,. \tag{7}$$

The ray parameter $p^{\min}$ is determined from Snell's law at the reflector by substituting $p_P = p_S = p^{\min}$. Note that, in contrast to the single-layer model, the normalized offset $x_{\min}/t_{\min}$ from equation (7) depends not only on the elastic parameters and reflector dip (Paper I), but also on the layer thicknesses.

Parameter estimation in layered VTI media

The analytic expressions from the previous section are valid if the dip plane of the reflector coincides with a vertical symmetry plane of all layers in the overburden. For example, if the medium is orthorhombic with two mutually orthogonal symmetry planes, our formalism can be used for two specific orientations of the reflector confined to the symmetry planes. (For any other reflector orientation, the 2-D equations are applicable only under the assumption of weak azimuthal anisotropy.) For azimuthally isotropic VTI media, however, there are no restrictions on the direction of reflector strike because each vertical plane is a plane of symmetry.

Below, we introduce a parameter-estimation algorithm for vertical transverse isotropy based on the parametric representation of the moveout of PS-waves and known expressions for the PS-wave NMO velocity from a horizontal reflector and the dip-dependent NMO velocity of P-waves. We start with an overview of the inversion algorithm for the single-layer model and proceed with a description of a two-stage moveout-inversion procedure for stratified VTI media.

Review of the single-layer algorithm

The goal of the parameter-estimation methodology introduced in Paper I is to invert the moveout of P and PS-waves from both a horizontal and a dipping reflector for the P- and S-wave vertical velocities (V_{P0} and V_{S0}) and the anisotropic coefficients ϵ and δ. P-wave reflection moveout in a VTI layer is fully governed by the parameters $V_{\mathrm{nmo},P}(0)$ and η [equations (1) and (2)] and, therefore, yields two equations for the medium parameters. The remaining information for the inversion is provided by the moveout of converted modes and the ratios of the zero-offset traveltimes of P- and PS-waves.

The moveout curve of the PS-wave from a horizontal reflector is symmetric with respect to zero offset because the VTI model has a horizontal symmetry plane. Furthermore, typically PS traveltimes on CMP spreads limited by reflector depth can be adequately described by the NMO velocity defined in the same way as for pure modes (Grechka, Theophanis and Tsvankin, 1999). NMO velocities of pure and converted modes in horizontally layered VTI media are related by the following Dix-type equation (Seriff and Sriram 1991; Tsvankin and Thomsen, 1994):

$$t_{PS0}\, V_{\text{nmo},PS}^2(0) = t_{P0}\, V_{\text{nmo},P}^2(0) + t_{S0}\, V_{\text{nmo},SV}^2(0)\,, \quad (8)$$

where t_{P0} and t_{S0} are the vertical traveltimes of the P and S-waves, and $t_{PS0} = t_{P0} + t_{S0}$. Hence, combining P and PS data allows us to determine the SV-wave NMO velocity $V_{\text{nmo},SV}(0)$ given (for a single VTI layer) by

$$V_{\text{nmo},SV}(0) = V_{S0}\sqrt{1+2\sigma}\,, \quad (9)$$

$$\sigma \equiv \left(\frac{V_{P0}}{V_{S0}}\right)^2 (\epsilon - \delta)\,. \quad (10)$$

Therefore, horizontal P and PS events in a VTI layer can be used to obtain the NMO velocities of P-waves [equation (1)] and SV-waves [equation (10)]. Also, the vertical-velocity ratio $\gamma \equiv V_{P0}/V_{S0}$ can be deduced from the vertical traveltimes of P- and PS-waves. This set of input data provides three constraints on four unknown medium parameters. Hence, for a given value of one of the parameters (e.g., δ), we can determine the other three from the horizontal events (Paper I):

$$V_{P0} = \frac{V_{\text{nmo},P}(0)}{\sqrt{1+2\delta}}\,, \quad (11)$$

$$V_{S0} = \frac{V_{P0}}{\gamma}\,, \quad (12)$$

$$\sigma = \frac{1}{2}\left[\frac{V_{\text{nmo},SV}^2(0)}{V_{S0}^2} - 1\right]\,, \quad (13)$$

$$\epsilon = \frac{\sigma}{\gamma^2} + \delta\,. \quad (14)$$

Next, the moveout of dipping events has to be inverted for the anisotropic coefficient δ. In principle, it seems to be sufficient to obtain the parameter η [equation (2)] using the P-wave NMO velocity from a dipping reflector. In this case, however, the parameter-estimation procedure is too unstable to be used in practice because small errors in η propagate with significant amplification into the vertical velocities (Paper I). A significant improvement can be achieved by including the moveout attributes of the PS-wave reflected from the same dipping interface. The attributes used in the single-layer problem are the slope of the moveout curve at zero offset, and, if the PS-wave traveltime has a minimum $t_{\min}(x_{\min})$ on the CMP gather, the normalized offset $x_{\min}/t_{\min}$ and the NMO velocity defined at $x = x_{\min}$. Numerical examples in Paper I confirm the accuracy and stability of this inversion methodology in the presence of realistic errors in input data.

Inversion in layered media.
Stage 1: CMP gathers

At the first stage of the inversion procedure, we estimate the interval values of the parameters V_{P0}, V_{S0}, ϵ and δ using P and PS data collected into CMP gathers. Suppose the model consists of two horizontal VTI layers intersected by a dipping interface (e.g., by a fault plane). After having determined the parameters of the subsurface layer using the algorithm described above, we proceed with the moveout inversion for the second layer. The first step is to perform layer-stripping of horizontal P and PS events to determine the interval NMO velocities of P and SV-waves. Combining P- and PS-wave NMO velocities for the reflection from the bottom of the second layer, we obtain the corresponding SV-wave NMO velocity from equation (8). Then the interval P- and SV-wave NMO velocities in the second layer are found using the conventional Dix differentiation (e.g., Tsvankin and Thomsen, 1994, 1995). The ratio of the interval vertical velocities in Layer 2 $\gamma^{(2)} \equiv V_{P0}^{(2)}/V_{S0}^{(2)}$ can be determined in a straightforward way from the zero-offset traveltimes of the P- and PS-waves.

Thus, the horizontal events yield the three interval parameters required by the single-layer inversion algorithm ($V_{\text{nmo},P}^{(2)}(0)$, $V_{\text{nmo},SV}^{(2)}(0)$ and $\gamma^{(2)}$). Using these results and equations (11)–(14), we can express the interval parameters of the second layer through the anisotropic coefficient $\delta^{(2)}$ exactly in the same way as in the single-layer problem. Then dipping P and PS events generated in the second layer can be inverted for $\delta^{(2)}$ and, therefore, for the full set of the interval parameters.

The layer-stripping of dipping P-wave events in layered VTI media is described in detail by Alkhalifah and Tsvankin (1995) and Alkhalifah (1997). Their algorithm makes it possible to find the interval NMO velocity $V_{\text{nmo},P}^{(2)}(p_{P0})$ of the dipping P-wave event in the second layer, where p_{P0} is the ray parameter of the zero-offset P-wave reflection from the dipping interface that can be determined from the reflection slope on the zero-offset section. To obtain $V_{\text{nmo},P}^{(2)}(p_{P0})$ from the effective NMO velocity of the dipping event, it is necessary to know two parameters of the subsurface layer – the zero-dip NMO velocity $V_{\text{nmo},P}^{(1)}(0)$ and the coefficient $\eta^{(1)}$.

As discussed above, the moveout of horizontal events and the NMO velocity $V_{\text{nmo},P}^{(2)}(p_{P0})$ have to be supplemented with the DMO attributes of the converted wave for estimating the medium parameters with sufficient accuracy. Our inversion algorithm operates with the two attributes – the slope of the moveout curve at zero offset ($dt/dx|_{x=0}$) and, for mild dips, the normalized offset of the traveltime minimum. (A more elaborate approach that uses PS traveltimes for a wider range of offsets is briefly outlined below.) To recover the moveout slope at zero offset, we approximate short-spread PS traveltimes with a shifted hyperbola, as discussed in Paper I. The coefficients of the hyperbola are found either by the least-squares method (if the traveltimes have been picked), or by semblance analysis of the seismograms.

Computation of the zero-offset moveout slope and the normalized offset of the traveltime minimum of the PS-wave for a layered VTI model using equations (4), (5) and (6) is described above. To define a "trial" model, we start with a certain value of the anisotropic coefficient $\delta^{(2)}$ and obtain the other parameters of the second layer using horizontal events. The dip of the reflector and the thickness of the second layer beneath the CMP ($z^{(2)}_{\rm CMP}$) – quantities needed to determine the moveout slope of the PS-wave – are calculated from the zero-offset traveltime of the dipping P event and the ray parameter p_{P0}. Hence, the horizontal events and the P-wave reflection from the dipping interface allow us to completely define the trial model that corresponds to the chosen value of $\delta^{(2)}$.

Finally, $\delta^{(2)}$ (and, consequently, the full parameter set of the second layer) is obtained by minimizing the objective function

$$\mathcal{F}_{\rm CMP} = \left[\frac{V_{{\rm nmo},P}(p_{P0}) - V^{\rm meas}_{{\rm nmo},P}(p_{P0})}{V^{\rm meas}_{{\rm nmo},P}(p_{P0})}\right]^2 + \left[\frac{(dt/dx|_{x=0}) - (dt/dx|_{x=0})^{\rm meas}}{(dt/dx|_{x=0})^{\rm meas}}\right]^2 + \left[\frac{(x_{\rm min}/t_{\rm min}) - (x_{\rm min}/t_{\rm min})^{\rm meas}}{(x_{\rm min}/t_{\rm min})^{\rm meas}}\right]^2, \qquad (15)$$

where the superscript "meas" denotes the measured values, and the quantities without a subscript are computed for a trial VTI model. The function (15) represents a system of three nonlinear equations with a single unknown parameter $\delta^{(2)}$. If the moveout curve of the PS-wave does not have a minimum on the CMP gather, the objective function includes only $V_{{\rm nmo},P}(p_{P0})$ and $dt/dx|_{x=0}$.

The dip moveout of the PS-wave is represented by either two or just one attribute. To increase the accuracy of the parameter estimation, we can include the whole conventional-spread CMP moveout of the PS-wave into the objective function. In this case, we generate the CMP gather of the PS-wave from equations (3), (4) and (5) for a given value of $\delta^{(2)}$ and find the rms difference between the modeled traveltimes and the "measured" moveout represented by the best-fit shifted hyperbola.

After the parameters of the second layer have been obtained, the parameter-estimation procedure continues downward in a layer-stripping fashion. It should be mentioned that the above operations with the horizontal events are entirely based on the Dix equation and, therefore, do not involve any information about the parameters of the subsurface layers. In contrast, the inversion of the P and PS traveltimes from a dipping reflector cannot be carried out without the parameter-estimation results for the overlying medium. If the overburden is known to be isotropic or elliptically anisotropic, its parameters (V_{P0}, V_{S0}, and $\epsilon = \delta$) can be extracted just from the moveout of horizontal P and PS events. In general VTI media, however, the layer-stripping procedure cannot be performed using horizontal events alone.

Numerical examples

The inversion algorithm designed for CMP gathers was tested on horizontally layered VTI models with through-going dipping interfaces. It was assumed that we can record both horizontal and dipping P and PS events for each layer, so that the input data would include the NMO velocities and vertical traveltimes of both modes from the horizontal reflectors, the NMO velocities and reflection slopes of the dipping P events, and the attributes of the dip moveout of the PS-wave. All these parameters were computed using the exact equations discussed above and distorted by Gaussian noise to simulate errors in the data. Then we applied the layer-stripping parameter-estimation algorithm based on the objective function (15) to 200 realizations of the noise-contaminated vector of input parameters.

For the three-layer model in Figure 3 the dips do not exceed 35°, and the objective function for the first two layers includes the normalized offset of the traveltime minimum. The scatter in the estimated parameters of the subsurface layer (Figures 3a,b) is the same as in the single-layer inversion results of Paper I. All four parameters (V_{P0}, V_{S0}, ϵ, δ) are recovered in a reasonably stable fashion, with the quasi-linear trends close to the lines corresponding to the correct values of η (Figure 3a) and V_{P0}/V_{S0} (Figure 3b). This is not surprising because η and V_{P0}/V_{S0} represent the parameter combinations most tightly constrained by the data. The velocity ratio V_{P0}/V_{S0} is determined directly from the vertical traveltimes, while η controls the P-wave NMO velocity from dipping reflectors.

The results for the second layer (Figures 3c,d) are comparable to those for Layer 1 because the subsurface layer is relatively thin, and the layer-stripping does not lead to a substantial error amplification. For the bottom layer (Figures 3e,f), however, the scatter in all four parameters becomes noticeably higher due to the distortions in the interval quantities produced by the layer-stripping procedure.

The model in Figure 4 contains a steeper through-going interface, with the dips in the 45–55° range. Comparison of Figures 3 and 4 shows that the the clouds of points become less elongated with increasing dip, which

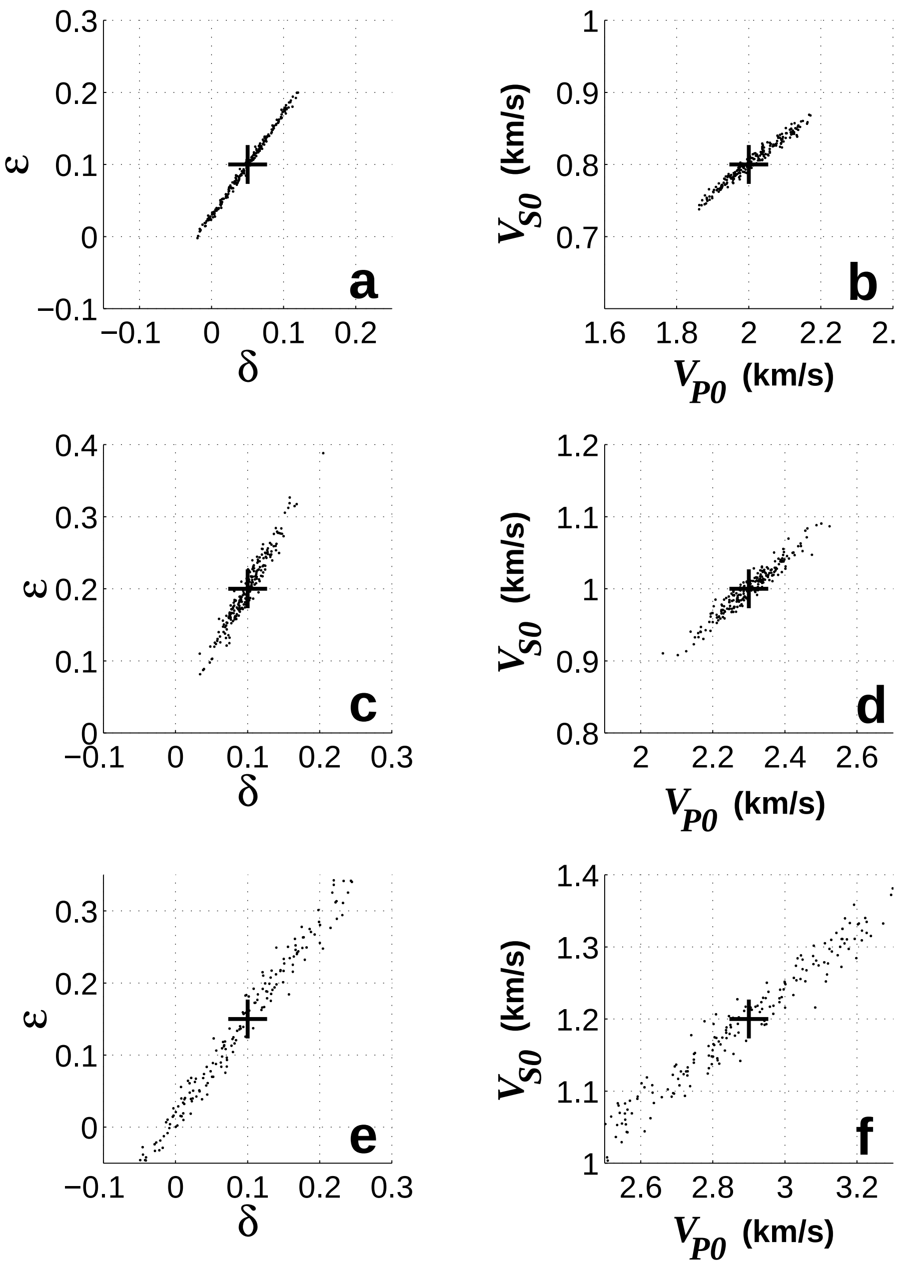

Figure 3. Interval parameters ϵ, δ, V_{P0}, and V_{S0} of a three-layer VTI medium determined by inverting P and PS moveout data from horizontal and dipping reflectors. The layer parameters are the same as in Figure 2; the dips of the interfaces in each layer are (from top to bottom) 30°, 35° and 30°. The input data were distorted by random noise with a standard deviation of 0.5% for the vertical traveltimes, 1.5% for the zero-dip NMO velocities and 2% for the moveout attributes of the dipping events. (a,b) – the results for the top layer (Layer 1); (c,d) – Layer 2; (e,f) – Layer 3; the actual model parameters are marked by the crosses.

signifies a more stable inversion procedure in all three layers. This result is explained by the higher sensitivity of the P-wave NMO velocity and PS-wave moveout attributes to the anisotropic parameters for larger dips. A similar influence of dip was observed for the single-layer model in Paper I.

Inversion in layered media. Stage 2: CRP gathers

If the reflector has a non-negligible curvature or an irregular shape at the scale of the CMP gather, the moveout of converted waves in CMP geometry may be distorted by reflection-point dispersal. Since equation (5) is de-

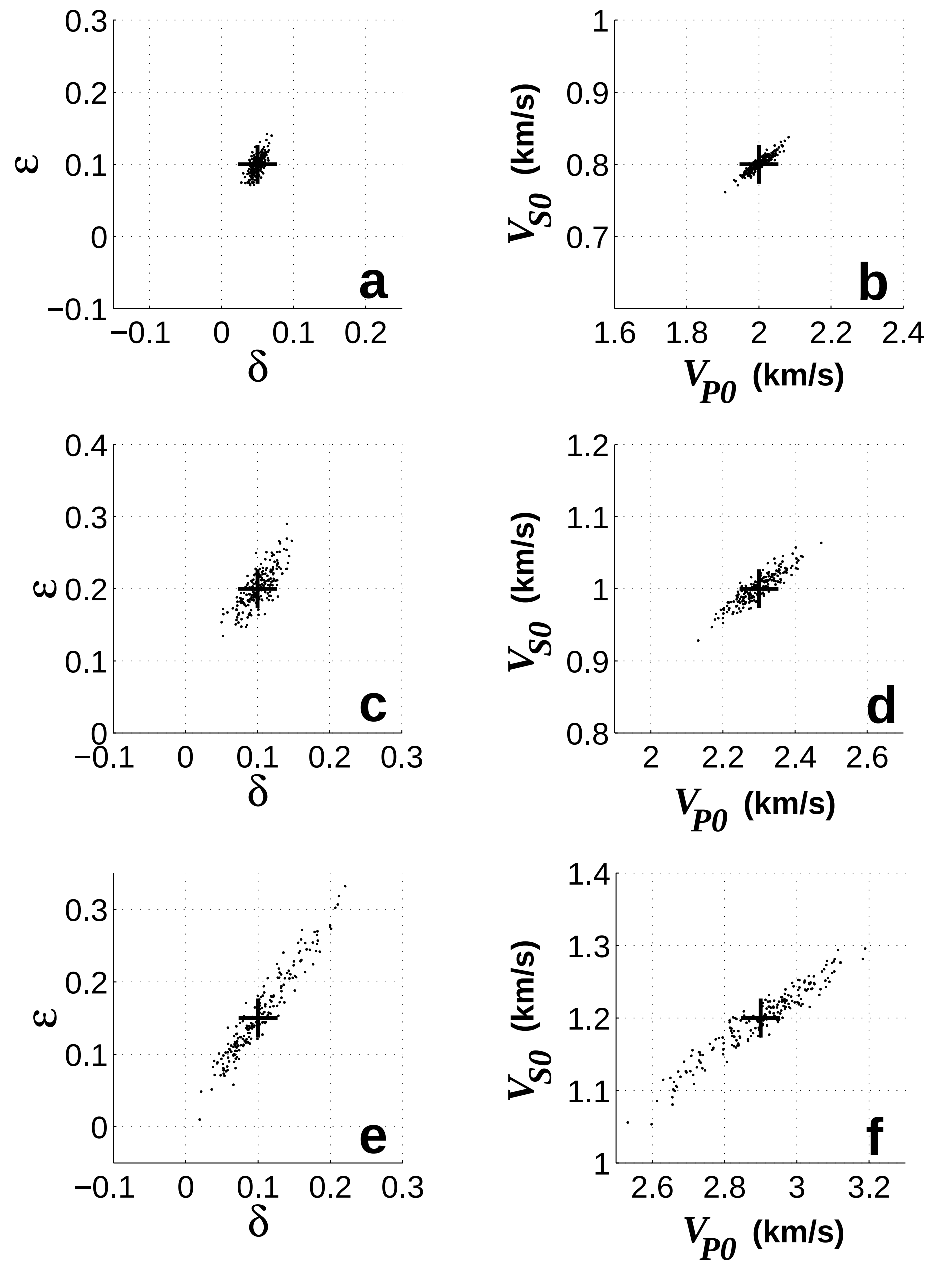

Figure 4. Inversion results for a three-layer VTI medium with steeper dipping interfaces than those in Figure 3. The parameters V_{P0}, V_{S0} ϵ and δ in each layer are the same as in Figures 2 and 3. The layer thicknesses are $z^{(1)} = 0.5$ km, $z^{(2)} = 1.0$ km and $z^{(3)} = 2$ km; the dips of the interfaces are: 45°, 50° and 55°. (a,b) – the results for Layer 1; (c,d) – Layer 2; (e,f) – Layer 3.

rived for a plane reflector, the inversion procedure in this case may produce inaccurate parameter estimates. To mitigate reflection-point dispersal, it is preferable to carry out both velocity analysis and processing of *PS*-waves on common-reflection-point (CRP) gathers. Resorting *PS* data into CRP gathers, however, is a costly procedure that has to be based on a known velocity model. While the lateral position of the reflection (conversion) point in isotropic media is controlled by the V_P/V_S ratio, in VTI media it is also sensitive to the anisotropic coefficients (Rommel, 1997; Thomsen, 1999). Therefore, velocity analysis on CRP gathers in our algorithm is preceded by the moveout inversion in CMP geometry described in the previous section.

Assuming that the inversion on CMP gathers yields a good approximation for the medium parameters, we search for the best-fit interval parameter $\delta^{(2)}$ only in a close vicinity of the obtained value. First, we resort the horizontal PS events into CRP gathers using the model determined at the first stage of the inversion. Semblance analysis on the CRP gathers gives updated values of the PS-wave NMO velocities from horizontal reflectors and, therefore, a more accurate estimate of the SV-wave velocity $V^{(2)}_{\text{nmo},SV}(0)$. With the updated $V^{(2)}_{\text{nmo},SV}(0)$ and the previously found values of $V^{(2)}_{\text{nmo},P}(0)$ and $\gamma^{(2)}$, we repeat the computation of the parameters of the second layer for a restricted range of $\delta^{(2)}$.

For each set of the medium parameters we reconstruct the dip and depth of the reflector in the second layer and resort the traces with the PS reflection from the dipping interface into CRP gathers. The common reflection point for the dipping PS event is chosen to coincide with the zero-offset reflection point of the dipping P-wave event because reflector dip is determined from the slope of the zero-offset P reflection. Then we generate the PS traveltimes on the CRP gather using equations (3) and (4) and determine the best-fit $\delta^{(2)}$ from the following objective function:

$$\mathcal{F}_{\text{CRP}} = \left[\frac{V_{\text{nmo},P}(p_{P0}) - V^{\text{meas}}_{\text{nmo},P}(p_{P0})}{V^{\text{meas}}_{\text{nmo},P}(p_{P0})}\right]^2 + t^{\text{rms}}_{PS}, \qquad (16)$$

where t^{rms}_{PS} is the rms difference between the modeled and measured traveltimes of the dipping PS event on the CRP gathers. In principle, it is possible to skip the inversion on CMP gathers and carry out the parameter-estimation for each trial value of $\delta^{(2)}$ directly on CRP gathers, but this algorithm is much more time consuming because it involves repeated resorting of PS data.

Discussion and conclusions

Most recent applications of converted waves were focused on improved imaging of targets poorly illuminated by P-wave data. In transversely isotropic media, PS-waves can also play an important role in parameter estimation because they are governed by the same anisotropic coefficients (ϵ and δ) as the P-waves. While P-wave reflection data contain enough information to carry out time-domain processing (NMO, DMO and time migration), they cannot be used to constrain the depth scale in horizontally layered VTI models above a dipping or horizontal reflector. Although the recent work of Le Stunff et al. (1999) indicates that for some piecewise homogeneous VTI media with *dipping* interfaces in the overburden P-wave moveout can be inverted for the vertical velocity, such an inversion still has to rely on a number of restrictive assumptions about the model.

In our previous paper on this subject (Tsvankin and Grechka, 1999; Paper I) we examined the joint inversion of reflection traveltimes of P- and PS-waves for the simplest model of a single VTI layer. We developed an analytic representation of the CMP moveout of mode conversions in the dip plane of the reflector and showed that all relevant VTI parameters (the coefficients ϵ and δ and the vertical velocities V_{P0} and V_{S0}) can be found by combining P and PS moveout data from a horizontal and a dipping reflector. An important conclusion of Paper I was that without the moveout attributes of the dipping PS event the inversion procedure is unstable and may give inaccurate values of the vertical velocities and reflector depth.

Here, we extend the results of Paper I to more realistic horizontally layered VTI media with throughgoing dipping reflectors. (The same model was adopted by Alkhalifah and Tsvankin (1995) for developing their well-known velocity-analysis method based on P-wave data.) Considering a CMP gather in the dip plane of the reflector, we express the traveltime and offset of converted waves in a parametric form through the slowness components of P- and S-legs of the reflected ray. After relating the P and S horizontal slownesses (ray parameters) via Snell's law at the reflector, we can compute the CMP traveltime curve without two-point ray tracing. This formalism also gives a concise description of such moveout attributes of the PS-wave, needed in the inversion procedure, as the slope of the moveout curve at zero offset and the normalized offset of the traveltime minimum.

Parameter estimation for stratified VTI media is performed by a layer-stripping algorithm operating solely with surface moveout data originally collected into CMP gathers. The interval NMO velocities of horizontal P and PS events, determined from the conventional Dix equation, are combined with the interval vertical traveltimes to give three equations for four unknown interval parameters (V_{P0}, V_{S0}, ϵ and δ). For a given interval value of δ, we obtain the interval V_{P0}, V_{S0} and ϵ from the horizontal events and fully reconstruct the trial model in the depth domain using the zero-offset traveltime and reflection slope of the dipping P event. Then we calculate the moveout attributes of the PS-wave and the interval NMO velocity of the dipping P event for the trial model and find the misfit between the computed and measured parameters. By minimizing the misfit (objective) function that contains a single unknown parameter (interval δ), we determine the full set of the interval parameters and continue the parameter-estimation procedure downward.

Converted-wave data in CMP geometry, however, may be corrupted by reflection-point dispersal on non-planar interfaces. Therefore, we suggest to refine the inversion results by resorting PS data into common-reflection-point (CRP) gathers and repeating the parameter-estimation procedure. Since the generation of the CRP gathers requires knowledge of the velocity model, it is preceded by the inversion of CMP moveout described above.

Our layer-stripping algorithm produces the depth distribution of the four parameters responsible for all signatures of P- and PS-waves in VTI media. Therefore, the inversion results can be used in prestack and poststack depth migration of P-wave data and processing of PS data for models with a stratified VTI overburden. The algorithm can be also applied without any modification in the vertical symmetry planes of layered orthorhombic media. Extension of this methodology to data acquired in wide-azimuth 3-D surveys will be discussed in a sequel paper.

Acknowledgments

We are grateful to Leon Thomsen (BP-Amoco) and members of the A(nisotropy)-Team of the Center for Wave Phenomena (CWP) at CSM for helpful suggestions. The support for this work was provided by the members of the Consortium Project on Seismic Inverse Methods for Complex Structures at CWP and by the United States Department of Energy by the United States Department of Energy (award #DE-FG03-98ER14908). I. Tsvankin was also supported by the Shell Faculty Career Initiation Grant.

References

Alkhalifah, T., 1997, Seismic data processing in vertically inhomogeneous TI media: Geophysics, **62**, 662-675.

Alkhalifah, T., Tsvankin, I., Larner, K., and Toldi, J., 1996, Velocity analysis and imaging in transversely isotropic media: Methodology and a case study: The Leading Edge, **15**, no. 5, 371-378.

Alkhalifah, T., and Tsvankin, I., 1995, Velocity analysis for transversely isotropic media: Geophysics, **60**, 1550–1566.

Anderson, J.E., Alkhalifah, T., and Tsvankin, I., 1996, Fowler DMO and time migration for transversely isotropic media: Geophysics, **61**, 835–844.

Grechka, V., Theophanis, S., and Tsvankin, I., 1999, Joint inversion of P- and PS-waves in orthorhombic media: Theory and a physical-modeling study: Geophysics, **64**, 146-161.

Grechka, V., Tsvankin, I., and Cohen, J.K., 1999, Generalized Dix equation and analytic treatment of normal-moveout velocity for anisotropic media: Geophys. Prosp., **47**, 117-148.

Grechka, V., and Tsvankin, I., 1998a, Feasibility of nonhyperbolic moveout inversion in transversely isotropic media: Geophysics, **63**, 957-969.

Grechka, V., and Tsvankin, I., 1998b, 3-D description of normal moveout in anisotropic inhomogeneous media: Geophysics, **63**, 1079-1092.

Le Stunff, Y., Grechka, V., and Tsvankin, I., 1999, Depth-domain velocity analysis in VTI media using surface P-wave data: Is it feasible?: this volume.

Rommel, B., 1997, Stacking charts for converted waves in a transversely isotropic medium: Center for Wave Phenomena Research Report (CWP-252).

Seriff, A.J., and Sriram, K.P., 1991, $P-SV$ reflection moveouts for transversely isotropic media with a vertical symmetry axis: Geophysics, **56**, 1271–1274.

Thomsen, L., 1986, Weak elastic anisotropy: Geophysics, **51**, 1954–1966.

Thomsen, L., 1999, Converted-wave reflection seismology over inhomogeneous, anisotropic media: Geophysics, in print.

Tsvankin, I., 1996, P-wave signatures and notation for transversely isotropic media: An overview: Geophysics, **61**, 467–483.

Tsvankin, I., and Grechka, V., 1999, Dip moveout of converted waves and parameter estimation in transversely isotropic media: Geophysical Prospecting, submitted.

Tsvankin, I., and Thomsen, L., 1994, Nonhyperbolic reflection moveout in anisotropic media: Geophysics, **59**, 1290–1304.

Tsvankin, I., and Thomsen, L., 1995, Inversion of reflection traveltimes for transverse isotropy: Geophysics, **60**, 1095–1107.

APPENDIX A: Converted-wave moveout from dipping reflectors in layered media

Here we extend the results of Paper I by considering common-midpoint reflection moveout of a PS-wave recorded in a vertical symmetry plane of *layered* anisotropic media. The vertical incidence plane is also assumed to coincide with the dip plane of the reflector, which makes the problem two-dimensional (i.e., both rays and the corresponding phase-velocity vectors of the reflected waves lie within the incidence plane). P-waves propagating in vertical symmetry planes are coupled to only the in-plane polarized shear wave (SV) and, therefore, generate a single (PSV) conversion.

The reflection traveltime in the layer immediately above the reflector can be written as (Figure 1)

$$t^{(\mathrm{N})} = t_P^{(\mathrm{N})} + t_S^{(\mathrm{N})} = \frac{z^{(\mathrm{N})}}{g_P^{(\mathrm{N})}\cos\theta_P^{(\mathrm{N})}} + \frac{z^{(\mathrm{N})}}{g_S^{(\mathrm{N})}\cos\theta_S^{(\mathrm{N})}}\,, \quad \text{(A1)}$$

where $\theta_P^{(\mathrm{N})}$ and $\theta_S^{(\mathrm{N})}$ are the angles between the P and S rays and vertical (Figure 1), $g_P^{(\mathrm{N})}$ and $g_S^{(\mathrm{N})}$ are the corresponding group velocities, and $z^{(\mathrm{N})}$ is the thickness of Layer N above the reflection point. The product $g^{(\mathrm{N})}\cos\theta^{(\mathrm{N})}$ for each mode is equal to the vertical component of the group-velocity vector $g_3^{(\mathrm{N})}$ that can be expressed through the components of the slowness vector as (Grechka et al., 1999)

$$\frac{1}{g_3} = q - p\, q_{,1}\,, \tag{A2}$$

where $p \equiv p_1$ and $q \equiv p_3$ are the horizontal and vertical slowness components, and $q_{,1} \equiv dq/dp_1$; hereafter, the subscript "1" in the slowness components will be omitted.

In each horizontal layer ℓ in the overburden, the traveltime can be found from equation (A1) with $z^{(\mathrm{N})}$ replaced by the layer thickness $z^{(\ell)}$. Note that since the medium above the reflector is laterally homogeneous, the horizontal slowness p (ray parameter) remains constant along both the P- and S-legs of the PS-ray. (The ray parameter does change at the conversion point, unless the reflector is horizontal.) Hence, the total reflection traveltime of the PS-wave is given by

$$t = \sum_{\ell=1}^{N} z^{(\ell)} \left(q_P^{(\ell)} - p_P\, q_{,P}^{(\ell)} + q_S^{(\ell)} - p_S\, q_{,S}^{(\ell)}\right). \tag{A3}$$

Here p_P and p_S are the ray parameters of the P- and S-waves related through Snell's law at the reflector, $q_{,P}^{(\ell)} \equiv dq_P^{(\ell)}/dp_P$ and $q_{,S}^{(\ell)} \equiv dq_S^{(\ell)}/dp_S$.

Next, we express the source-receiver offset x in terms of the slowness components. For the horizontal displacement of the P-wave ray in layer ℓ we have (Paper I)

$$x_P = z^{(\ell)} \tan\theta_P^{(\mathrm{N})} = z^{(\ell)}\, \frac{g_{1P}}{g_{3P}} = -z^{(\ell)}\, q_{,P}^{(\ell)}\,. \tag{A4}$$

Summing over the stack of layers yields the total horizontal ray displacement with respect to the reflection point:

$$x_P = -\sum_{\ell=1}^{N} z^{(\ell)}\, q_{,P}^{(\ell)}\,. \tag{A5}$$

An analogous expression is valid for the S-wave ray. The source-receiver offset is determined by the difference between the horizontal displacements of the S- and P-rays:

$$x = x_S - x_P = \sum_{\ell=1}^{N} z^{(\ell)} \left(q_{,P}^{(\ell)} - q_{,S}^{(\ell)}\right). \tag{A6}$$

Since the horizontal coordinate axis x_1 is assumed to run updip (Figure 1), taking x_P in equation (A6) with a negative sign implies that the offset x is positive if the P-leg is located *downdip* with respect to the S-leg.

To find an analytic representation of a common-midpoint (CMP) gather, we need to replace the thickness of layer N above the reflection point ($z^{(\mathrm{N})}$) by the corresponding thickness beneath the common midpoint ($z_{\mathrm{CMP}}^{(\mathrm{N})}$; see Figure 1). If we denote the horizontal distance between the CMP and reflection point as Δx, $z_{\mathrm{CMP}}^{(\mathrm{N})}$ can be expressed as

$$z_{\mathrm{CMP}}^{(\mathrm{N})} = z^{(\mathrm{N})} + \Delta x\, \tan\phi\,, \tag{A7}$$

where ϕ is reflector dip. Taking into account the signs of the horizontal displacements of the P- and S-rays in equations (A5) and (A6), we find

$$\Delta x = \frac{-x_P - x_S}{2} = \frac{1}{2} \sum_{\ell=1}^{N} z^{(\ell)} \left(q_{,P}^{(\ell)} + q_{,S}^{(\ell)}\right). \tag{A8}$$

Substituting equation (A8) into equation (A7) and separating $z^{(\mathrm{N})}$ in Δx yields

$$z_{\mathrm{CMP}}^{(\mathrm{N})} = z^{(\mathrm{N})} + \frac{\tan\phi}{2} \sum_{\ell=1}^{N-1} \left[z^{(\ell)} \left(q_{,P}^{(\ell)} + q_{,S}^{(\ell)}\right)\right] + \frac{\tan\phi}{2}\, z^{(\mathrm{N})} \left(q_{,P}^{(\mathrm{N})} + q_{,S}^{(\mathrm{N})}\right). \tag{A9}$$

Equation (A9) can be solved for $z^{(\mathrm{N})}$:

$$z^{(\mathrm{N})} = \frac{z_{\mathrm{CMP}}^{(\mathrm{N})} - \frac{\tan\phi}{2} \sum_{\ell=1}^{N-1} \left[z^{(\ell)} \left(q_{,P}^{(\ell)} + q_{,S}^{(\ell)}\right)\right]}{1 + \frac{\tan\phi}{2} \left(q_{,P}^{(\mathrm{N})} + q_{,S}^{(\mathrm{N})}\right)}\,. \tag{A10}$$

Equation (A10) makes it possible to express the traveltime and offset of the converted wave [equations (A3) and (A6)] through the slowness components of the P- and S-waves and the thickness of Layer N beneath the common midpoint.