

Short Note:

Estimation of anisotropy from multi-azimuth walkaway VSP data in the presence of lateral heterogeneity

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ABSTRACT

Multi-azimuth, walkaway, vertical seismic profiling (VSP) is often used to obtain in situ slowness surfaces and anisotropic parameters. Conventionally, processing of VSP data is performed under the assumption of lateral homogeneity of the subsurface, which makes it possible to replace the horizontal slowness components at receiver location in the borehole by those measured at the surface. The presence of either lateral velocity variations or dips of intermediate interfaces, however, leads to distortions in the reconstructed slowness surfaces and, consequently, to inaccuracies in the estimated anisotropic parameters. Although the origin of those errors is clear, it is known to be difficult to remove them.

Here we suggest to combine multi-azimuth VSP data with surface reflection seismics. We show that tracing rays in a 3-D isotropic macro-velocity model obtained from seismic processing is usually sufficient to account for the influence of laterally heterogeneous overburden on VSP data. This result can be explained by ray theory which implies that only lateral variations in *anisotropy* (which are normally small) cannot be removed using our approach.

Key words: multi-azimuth walkaway VSP, lateral velocity heterogeneity, anisotropic parameter estimation

Introduction

Accounting for lateral velocity variation has been a long-standing obstacle in using traveltimes measured in VSP geometry for characterizing in situ anisotropy. If lateral homogeneity of the subsurface above downhole receivers is assumed, Snell's law implies that the horizontal slowness components

$$p_i = \frac{\partial t}{\partial x_i}, \quad (i = 1, 2), \quad (1)$$

obtained *at the surface* as the derivatives of traveltimes t with respect to the source coordinates x_i , are exactly equal to those *at the receiver depth*. Along with the vertical slowness components

$$q = \frac{\partial t}{\partial z}, \quad (2)$$

the values of p_i are sufficient for reconstructing the slow-

ness surface $q(p_1, p_2) = 0$ at the receiver depth z . This straightforward scheme, however, breaks down if the overburden is laterally heterogeneous. The reason is simple: the horizontal slowness $\mathbf{p} = [p_1, p_2]$ varies along each ray and, therefore, the values measured at the surface no longer equal to those at depth z . Thus, the actual functional dependence $q(p_1, p_2)$ gets distorted which, in turn, leads to errors in the estimated anisotropic parameters. Gaiser (1990) and Sayers (1997) presented a series of numerical tests and showed that even when lateral velocity variation seems to be insignificant, such as in the case of dipping interfaces in the overburden with dips of about 5° , the resulting distortions of the slowness surface $q(p_1, p_2)$ are noticeable.

Grechka, Tsvankin, and Contreras (1999) attempted to correct for weak lateral heterogeneity assuming that the anisotropy is factorized (i.e., velocity varies laterally,

whereas anisotropic coefficients do not), and the subsurface is vertically homogeneous in each interval between adjacent groups of downhole receivers. Their approach lead to a layer-stripping type technique which turned out to be unstable in the realistic case when velocity changes both laterally and vertically. The underlying theoretical reason for this result is that traveltimes $t(x_1, x_2, z)$ at a fixed depth z do not allow one to distinguish between lateral and vertical heterogeneity of the overburden. Thus, some other information has to be added to separate those two types of velocity variation. Such information might be obtained from conventional processing of surface reflection data which are usually available at the time when VSP is performed. We show that a reasonable approximation for the horizontal slowness components at the receiver depth can be obtained by tracing rays through the reconstructed *isotropic* macro-velocity model even though the subsurface is anisotropic. We start with theoretical justification of this fact and then present a numerical example that validates our method.

Theoretical considerations

Let us denote p_i^s the horizontal slowness components obtained by differentiating the recorded traveltime [equation (1)] at a certain source location x_i . Due to the possible presence of lateral heterogeneity, the slowness p_i^s changes along the ray by the amount p_i^Δ and becomes $p_i^r = p_i^s + p_i^\Delta$, (i = 1, 2) (3)

at the receiver. The quantities p_i^Δ are given by the ray-tracing equations (e.g., Červený, Molotkov, and Pšenčík, 1977):

$$p_i^\Delta = -\frac{1}{2} \int \frac{\partial c_{jklm}}{\partial x_i} A_j p_k p_l A_m d\tau, \quad (4)$$

where integration with respect to time τ is performed along the ray. In equation (4), c_{jklm} is the density-normalized stiffness tensor, \mathbf{p} is the slowness vector, \mathbf{A} is the unit polarization vector, and the summation from 1 to 3 over all repeated indices is assumed. The main practical difficulty in evaluating integral (4) and obtaining the slowness correction p_i^Δ is that the stiffness tensor

$$\mathbf{c} = \mathbf{c}(\mathbf{x}) \equiv c_{jklm}(\mathbf{x}) \quad (5)$$

is unknown.

Even though we generally do not know \mathbf{c} , its “isotropic” component \mathbf{c}^{iso} can be obtained from conventional processing of reflection data. Thus, we can write

$$\mathbf{c} = \mathbf{c}^{\text{iso}} + \mathbf{c}^{\text{ani}}, \quad (6)$$

where \mathbf{c}^{ani} is the “anisotropic” portion of \mathbf{c} which is generally unknown. Equation (6) separates the stiffness tensor \mathbf{c} into its isotropic and anisotropic parts. Clearly, \mathbf{c}^{ani} is zero in isotropic media.

The strength of the anisotropy can be measured by dimensionless anisotropic coefficients ϵ defined as ratios of certain components (or their combinations) of tensor \mathbf{c} (e.g., Thomsen, 1986; Tsvankin, 1997). Although the anisotropic coefficients have proven extremely useful for a wide variety of seismic problems, we will use equation (6) in the derivations below because it leads to more intuitive and simple results. Since the absolute values of anisotropic coefficients ϵ rarely exceed 0.2–0.3, one might expect that the inequality

$$\|\mathbf{c}^{\text{iso}}\| > \|\mathbf{c}^{\text{ani}}\| \quad (7)$$

for the norms of tensors \mathbf{c}^{iso} and \mathbf{c}^{ani} is usually satisfied. Therefore, a significant portion of lateral velocity variation, which is incorporated into \mathbf{c}^{iso} , can be estimated from surface seismics.

Next, substituting equation (6) into (4), we obtain

$$p_i^\Delta = -\frac{1}{2} \int \frac{\partial (\mathbf{c}^{\text{iso}} + \mathbf{c}^{\text{ani}})}{\partial x_i} \cdot \mathbf{B} d\tau, \quad (8)$$

where the tensor

$$\mathbf{B} \equiv A_j p_k p_l A_m, \quad (9)$$

and the dot denotes summation from 1 to 3 over all indices of the tensors $\partial (\mathbf{c}^{\text{iso}} + \mathbf{c}^{\text{ani}}) / \partial x_i$ and \mathbf{B} .

Further rewriting equation (8) in the form

$$p_i^\Delta = -\frac{1}{2} \int \frac{\partial \mathbf{c}^{\text{iso}}}{\partial x_i} \cdot \mathbf{B} d\tau - \frac{1}{2} \int \frac{\partial \mathbf{c}^{\text{ani}}}{\partial x_i} \cdot \mathbf{B} d\tau \quad (10)$$

gives us a recipe for obtaining a useful approximation for p_i^Δ . Dropping the second term on the right-hand side of equation (10) yields

$$p_i^\Delta \approx -\frac{1}{2} \int \frac{\partial \mathbf{c}^{\text{iso}}}{\partial x_i} \cdot \mathbf{B} d\tau, \quad (i = 1, 2). \quad (11)$$

This integral can be easily evaluated by tracing rays through the isotropic velocity model \mathbf{c}^{iso} reconstructed from reflection seismics. The initial directions of the rays are specified by the horizontal slowness components p_i^s measured from VSP data.

The second (ignored) term in equation (10) describes the influence of *laterally varying anisotropy* on the slowness correction p_i^Δ . Since anisotropy is usually relatively small by itself [see inequality (7)] and its laterally changing component is probably even smaller, the contribution of the neglected term is expected to be insignificant. This result is intuitively obvious because laterally homogeneous anisotropic overburden does not change p_i .

Although the ray-tracing theory suggests that the correction (11) may be sufficient, the above qualitative discussion does not give us any estimate of the errors caused by this correction. To quantify these errors, we present a numerical test.

Numerical example

Here we show that lateral heterogeneity in a model of moderate complexity (Figure 1) might produce significant errors in estimates of anisotropy at the receiver level. We also demonstrate that the above described correction removes most of the influence of laterally heterogeneous overburden.

Our model (Figure 1) contains four homogeneous transversely isotropic layers with a vertical symmetry axis (VTI media). The relevant layer parameters – the vertical P -wave velocities V_{P0} and Thomsen (1986) anisotropic coefficients ϵ and δ – are given in Table 1. To generate VSP data, we traced P -wave rays between 6561 surface sources and five downhole receivers. The sources are evenly distributed over a square area of the 4×4 km size; the source increments are 50 m. The borehole with geophones at depths 1.970, 1.985, 2.000, 2.105, and 2.030 km is located at the center of the square and has coordinates $x_1 = x_2 = 0$ (Figure 1).

Lateral heterogeneity in our model is associated with irregular interfaces in the overburden. The heterogeneity clearly manifests itself on the traveltimes contours shown in Figure 2. Not only the traveltime minimum is shifted from the borehole location by approximately 0.2 km but also the traveltime varies with azimuth, whereas it should be azimuthally independent with the minimum at borehole location for laterally homogeneous VTI layers. As discussed by Grechka et al. (1999), the shift of traveltime minimum can be caused either by the absence of the horizontal symmetry plane in some anisotropic layers or by lateral velocity variation in the overburden. In our model it is the latter.

If one performed naive VSP processing using uncorrected horizontal slowness components p_1^s and p_2^s , which can be obtained directly from Figure 2, it would produce the slowness surface $q(p_1^s, p_2^s)$ shown in Figure 3a. The presence of lateral heterogeneity in the overburden leads to the surface that corresponds to some *azimuthally* anisotropic medium *without* a horizontal symmetry plane, as indicated by the shift of the maximum value of q from the point $p_1^s = p_2^s = 0$. In contrast, the contours of correct surface $q(p_1^r, p_2^r)$ are circles centered at $p_1^r = p_2^r = 0$ (Figure 3b), as one would expect for VTI media.

We can try to find a model that fits the slowness surface $q(p_1^s, p_2^s)$ in Figure 3a. Since this model has to be azimuthally anisotropic without a horizontal symmetry plane, we have chosen to invert the slowness $q(p_1^s, p_2^s)$ for a tilted orthorhombic medium. This yields the set of Tsvankin's (1997) anisotropic parameters listed in Table 2. Analyzing the results, we conclude:

(1) The medium obtained is indeed orthorhombic because the anisotropic coefficients $\epsilon^{(1)}$ and $\epsilon^{(2)}$, and $\delta^{(1)}$

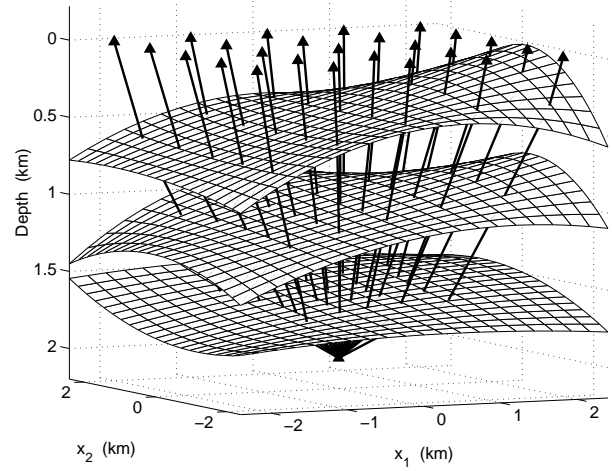


Figure 1. Rays in VSP geometry recorded by a buried receiver at depth 2 km.

Layer	V_{P0} (km/s)	ϵ	δ
1	2.00	0.10	0.05
2	2.80	0.25	0.05
3	3.10	0.15	0.10
4	4.40	0.15	0.05

Table 1. Parameters of the VTI model shown in Figure 1.

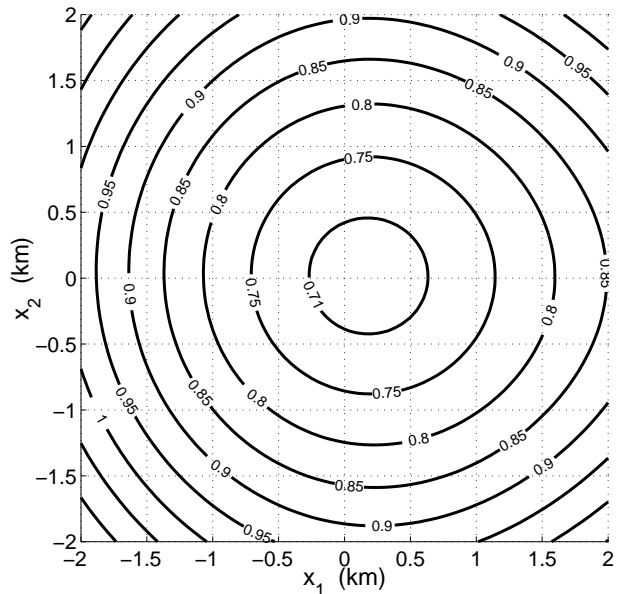


Figure 2. P -wave traveltimes (in s) computed for the receiver located at $x_1 = x_2 = 0$, $z = 2$ km.

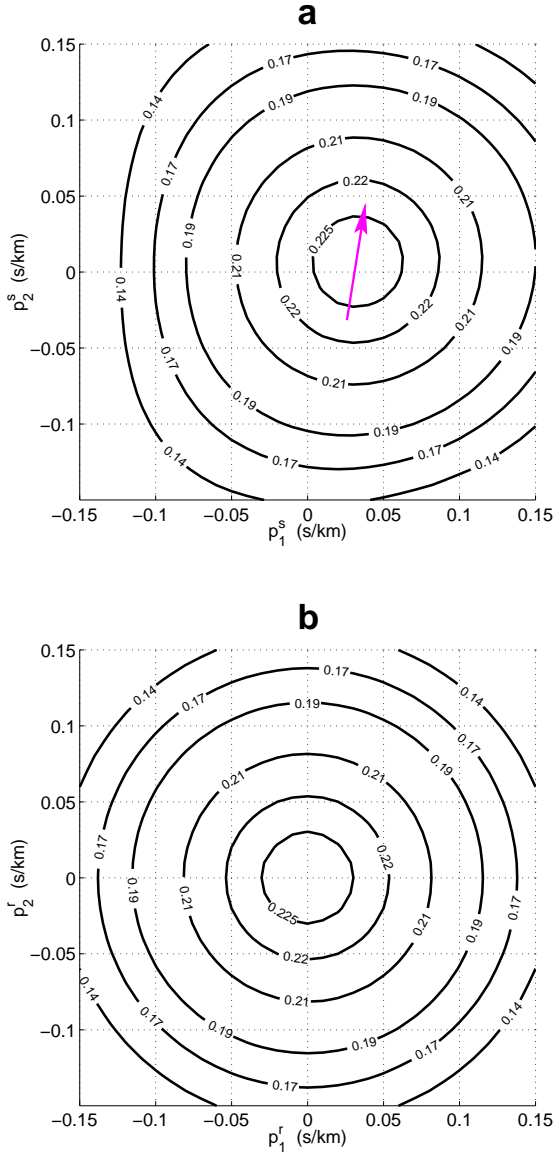


Figure 3. Contours of the slowness surfaces at the depth $z = 2$ km: (a) $q(p_1^s, p_2^s)$ and (b) $q(p_1^r, p_2^r)$. The horizontal slowness components p_1^s and p_2^s correspond to traveltimes in Figure 2; the components p_1^r and p_2^r are taken at the receiver location.

V_{P0}	$\epsilon^{(1)}$	$\delta^{(1)}$	$\epsilon^{(2)}$	$\delta^{(2)}$	$\delta^{(3)}$	β
5.21	-0.02	-0.19	-0.19	-0.25	0.15	50.1

Table 2. Tsvankin's parameters of the best-fit orthorhombic medium estimated by inverting the slowness surface $q(p_1^s, p_2^s)$ (Figure 3a). β is the tilt (in degrees with respect to horizontal) of the axis x_3 required to produce the asymmetric slowness contours $q(p_1^s, p_2^s)$. The angle β is measured clockwise in the vertical plane orthogonal to the direction marked with arrow in Figure 3a. V_{P0} is the velocity (in km/s) along the tilted x_3 -axis.

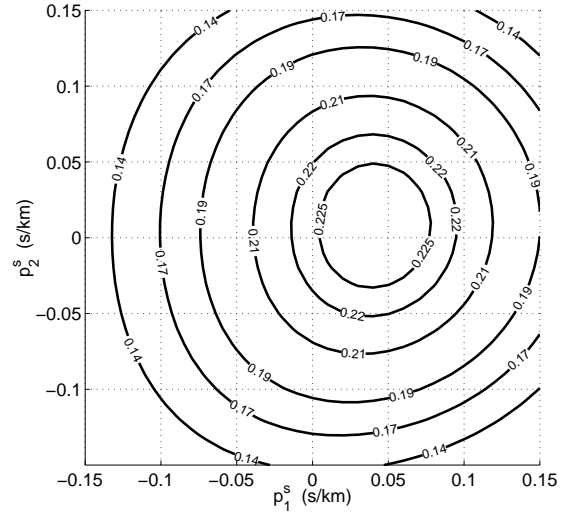


Figure 4. Slowness contours obtained for the reconstructed orthorhombic model with parameters given in Table 2.

and $\delta^{(2)}$, which should be equal in VTI media ($\epsilon^{(1)} = \epsilon^{(2)}$ and $\delta^{(1)} = \delta^{(2)}$), significantly deviate from each other. Also, the coefficient $\delta^{(3)}$ does not vanish, as it does in VTI media.

(2) The tilt $\beta = 50.1^\circ$ of the x_3 -axis is necessary to fit the asymmetric slowness contours in Figure 3a.

To show that the orthorhombic medium described by the anisotropic parameters given in Table 2 does fit the slowness surface $q(p_1^s, p_2^s)$ in Figure 3a, we plotted the reconstructed slowness in Figure 4. Apart from the difference in the contour lines $q = 0.225$ and $q = 0.22$ s/km, all the other contours look very similar. Thus, we were able to compensate for the influence of lateral velocity variation on the slowness surface by artificially introducing azimuthal anisotropy. Clearly, our naive method of estimating anisotropy at the receiver depth leads to substantial errors. To improve the results, we use the more elaborate approach outlined in the previous section.

First, we compute P -wave reflection traveltimes corresponding to all three model interfaces (Figure 1) and, simulating conventional processing, invert our traveltimes data for the best-fit isotropic model. The details of the inversion procedure applied here are described by Grechka et al. (2000). Table 3 lists the interval velocities obtained for layers 1, 2, and 3; the velocity below the third reflector, estimated by fitting a sphere to the slowness contours in Figure 3a, is $V_{P,4} = 4.43$ km/s. The velocities in the overburden are higher than the corresponding vertical velocities in Table 1. This is not surprising. Since the dips of reflectors in our model are mild, and the isotropic velocities $V_{P,\ell}$ ($\ell = 1, 2, 3$) were estimated from

$V_{P,1}$	$V_{P,2}$	$V_{P,3}$
2.12	2.98	3.38

Table 3. Interval velocities (in km/s) in the isotropic model obtained from P -wave reflection data.

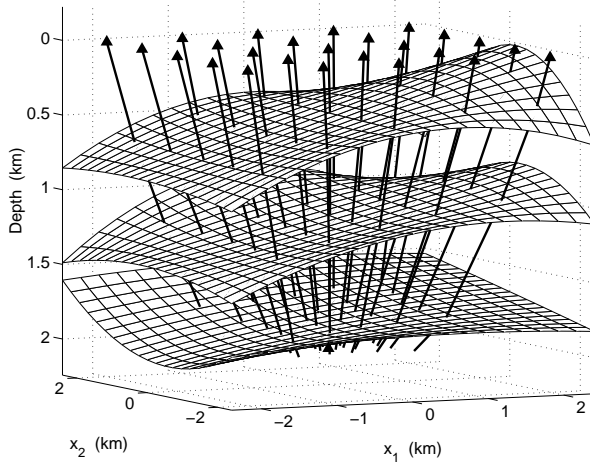


Figure 5. Rays traced through the reconstructed isotropic model.

the stacking velocities, one should expect that (Thomsen, 1986)

$$V_{P,\ell} \approx V_{P0,\ell} (1 + \delta_\ell), \quad (12)$$

where $V_{P0,\ell}$ and δ_ℓ are the interval parameters from Table 1. Comparing the quantities in Table 1 and 3 shows that the approximation (12) is sufficiently accurate.

Our second step is to trace rays from the surface sources to the receiver level through the reconstructed isotropic model. The initial ray directions are specified by the slowness components p_1^s and p_2^s at the source locations. Several such rays, terminated at the receiver depth 2 km, are shown in Figure 5. An immediate observation made from this plot is that the rays do *not* hit the receiver. They miss it by as much as 1.27 km, which is close to one-half of the maximum horizontal distance between the sources and the borehole. A detailed comparison of the interfaces in Figures 1 and 5 reveals that not only their shapes are different, but also the interfaces in Figure 5 are generally deeper. The latter is the consequence of ignoring Thomsen δ coefficient in the isotropic velocity analysis. Thus, we can conclude that our reconstructed isotropic model is incorrect. Although it can be improved by doing the VTI inversion as described by Grechka et al. (2000), the model parameters in the depth domain cannot be estimated unambiguously because the dips of

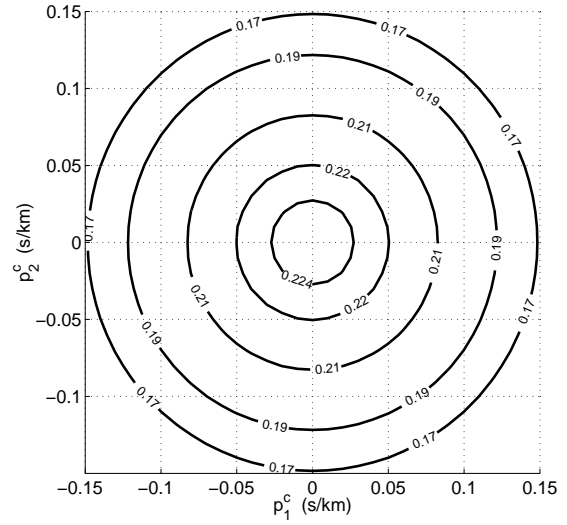


Figure 6. Contours of the slowness surface $q(p_1^c, p_2^c)$ obtained by tracing rays through the reconstructed isotropic model.

V_{P0}	$\epsilon^{(1)}$	$\delta^{(1)}$	$\epsilon^{(2)}$	$\delta^{(2)}$	$\delta^{(3)}$	β
4.40	0.12	0.04	0.15	0.05	-0.02	-0.9

Table 4. Tsvankin's parameters of the best-fit orthorhombic medium and the tilt β (in degrees) of the x_3 -axis estimated by inverting the slowness surface $q(p_1^c, p_2^c)$ shown in Figure 6.

the interfaces are too mild to constrain the parameters in a unique fashion.

Our goal, however, is *not* to obtain the correct subsurface model but, rather, to account for the influence of lateral velocity variations on the horizontal slowness components. This can be achieved by constructing the new slowness surface $q(p_1^c, p_2^c)$ (Figure 6) from the slowness components p_i^c that correspond to the rays traced down to the receiver level. Clearly, the surface in Figure 6 is much closer to the correct slowness $q(p_1^r, p_2^r)$ (Figure 3b) compared to the previously obtained surface $q(p_1^s, p_2^s)$ (Figure 3a). Fitting a tilted orthorhombic model to the slowness $q(p_1^c, p_2^c)$ produces the set of anisotropic parameters given in Table 4. Now we were able to reconstruct anisotropy at the receiver depth with a significantly higher accuracy. Although the obtained medium is formally orthorhombic, the deviation of its anisotropic coefficients from the parameters of the actual VTI model ($\epsilon^{(1)} = \epsilon^{(2)} = 0.15$, $\delta^{(1)} = \delta^{(2)} = 0.05$, and $\delta^{(3)} = 0$; see the bottom line of Table 1) does not exceed 0.03. In addition, the small value of the tilt $\beta = -0.9^\circ$ indicates that the best-fit orthorhombic medium has a nearly horizontal symmetry plane.

Overall, the numerical example presented illustrates that the proposed correction of VSP data for lateral ve-

locity heterogeneity produces useful results in the presence of moderate anisotropy in the overburden. Thus, the test has confirmed our expectations that were based on general ray-theoretical considerations.

Discussion and conclusions

We described a method for solving the well-known problem of correcting the slowness surfaces, reconstructed from multi-azimuth walkaway VSP data, for the influence of lateral heterogeneity in the overburden. We approached this problem by recognizing that reflection seismics, which is sufficiently sensitive to lateral velocity variations, might provide important information about the velocity field that cannot be inferred from VSP data alone. We showed that this information can be utilized by performing ray tracing through the velocity model that results from conventional (isotropic) seismic processing. Although anisotropy in the overburden is ignored in our correction, this does not lead to substantial distortions of the reconstructed slowness surfaces because the errors in horizontal slowness are caused only by the *laterally varying* portion of the anisotropy. This conclusion, first derived from the ray theory, is confirmed by the numerical example presented in the paper.

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